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CHARGE DISTRIBUTION AND STRUCTURE OF INHOMOGENEOUS PLASMA CONFINED BY UNLIKE-CHARGE PLANES

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Density and charge distributions in an equilibrium plasma confined by unlike-charge planes are determined in which the potential energy in the middle field may be comparable to the kinetic one. It is shown that in such a system, provided it is not too narrow ($h/D > 2.6$) and the intensity of the external field is sufficiently high ($eED/kT > 1.5$), a three-layer structure of plasma develops: near boundaries - a layer of practically one-component plasma of the corresponding species; at the center - a layer of almost electroneutral plasma. A further increase of the field intensity leads to degeneracy of the electroneutral plasma layer.

1. An inhomogeneous equilibrium state of plasma and plasma-like media is encountered in a number of various situations, among which noteworthy are the plasma layers being formed near surfaces of electrostatic probes [1], the plasma with a condensed disperse phase [2] as well as the spatially confined plasmoids carrying an uncompensated electric charge or being in an external electrostatic field. Near the boundaries of such plasma the narrow regions of localization of an electric field E_c come into play [3, 4]; in this case a situation may occur where $eE_c\delta > kT$, with δ being the characteristic dimension of the region where the field is localized. Just in that situation the plasma becomes very inhomogeneous and its strong inhomogeneity state does not contradict the condition of its weak non-ideality, implying a small value of the plasma parameter γ at any point of the space occupied by the plasma [3]:

$$\gamma = \frac{e^2}{4\pi\varepsilon_0 kTD_l} \ll 1, \quad D_l^{-2} \equiv \frac{e^2}{\varepsilon_0 kT} \sum_\alpha n_\alpha^l Z_\alpha^2, \tag{1}$$

where n_α^l is the local numerical density of particles of the species α.

In a system containing unlike-charge particles, in addition to the Coulomb interaction it is also necessary to take into account short-duration repulsion. In [5], it is shown that although there are deep potential wells $\varepsilon \gg kT$, the character of the collective interaction of particles changes and if the criterion

$$\zeta\left(\frac{\varepsilon}{kT}\right) < \gamma^{-1} \tag{2}$$

is fulfilled, it is sufficient to employ the pure Debye approach to study distribution functions at distances larger than the Landau length. The function $\zeta(\varepsilon/kT)$ is tabulated in [5] for the model of solid spheres and it has turned out that condition (2) is not strict.

The present work is devoted to the investigation of the charge distribution in the plane-parallel layer of an equilibrium classical highly inhomogeneous but weakly nonideal two-component charge-symmetric plasma confined by unlike-charge planes. Fulfillment of conditions (1), (2) justifies application of the Debye uncoupling of the chain of the BBGKI (Bogolyubov-Borhn-Green-Kirkwood-Ivon) equations [6]. In the lower order of the perturbation theory for the plasma parameter γ the first equation in the BBGKI chain for a two-dimensional section of the surface for the system under discussion becomes [7] as follows

$$\nabla_1 \ln F_\alpha(1) - \frac{eZ_\alpha E}{kT} + \frac{e^2 Z_\alpha}{4\pi\varepsilon_0 kT} \sum_\beta n_\beta Z_\beta \int\limits_{-\frac{h}{2} \leqslant x \leqslant \frac{h}{2}} F_\beta(2) \nabla_1 \frac{1}{r_{12}} d2 = 0. \tag{3}$$

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The field **E** is external, homogeneous in virtue of plane symmetry, and directed along the 0x axis perpendicular to the plasma boundary [8]. Besides, owing to the plane symmetry the one-particle distribution functions depend only on the x-coordinate of the particles. In virtue of the bilateral symmetry relative to the plane X = 0 equidistant from the boundaries of the system and the charge symmetry ($Z_+ = -Z_-$)

$$F_+(x) = F_-(-x), \tag{4}$$

where $F_+(x)$ and $F_-(x)$ are the one-particle distribution functions of particles of the corresponding species.

2. Consider the behavior of the one-particle distribution function $F_\alpha(x)$ in the lower, with respect to γ, approximation neglecting pair correlations. From system of equations (3) with an account of condition (4) we find the relationship between the distribution functions of particles of different species:

$$F_+(x) F_-(x) = F_0^2, \tag{5}$$

where $F_0 \equiv F_+(0)$. For definiteness, we assume that the right-hand boundary is negatively charged, so that the external field E is 0x-axis codirected. System (3) with an account of (5) is identically transformed to the form

$$\ln' f(x) = \frac{1}{D^*} \sqrt{\frac{f^2(x) - (2-\rho^2) f(x) + 1}{f(x)}}, \tag{6}$$

where $f(x) \equiv F_+(x)/F_0$; $\rho \equiv D^* f'(x)|_{x=0}$; $D^* \equiv D/\sqrt{F_0}$, while the Debye radius

$$D^{-2} = \frac{e^2 n}{\varepsilon_0 kT}$$

is determined in terms of the mean numerical density of particles n within the entire space of plasma.

If

$$|\rho| < 2, \tag{7}$$

then the square trinomial under the radical sign in (6) has complex roots and the solution of Eq. (6) in terms of elliptical functions [9] is

$$f_\pm(x) = 2\frac{\mathrm{dn}^2(X;\ \lambda)}{\mathrm{cn}^2(X;\ \lambda)} - 1 \pm F_0\rho \frac{\mathrm{sn}(X;\ \lambda)\,\mathrm{dn}(X;\ \lambda)}{\mathrm{cn}^2(X;\ \lambda)}, \tag{8}$$

where $X \equiv x/D^*$, while λ is the elliptic integral modulus

$$\lambda^2 = 1 - \frac{1}{4}\rho^2. \tag{9}$$

The local density of the charge $q(x) \equiv e\sum_\alpha n_\alpha Z_\alpha F_\alpha(x)$ is

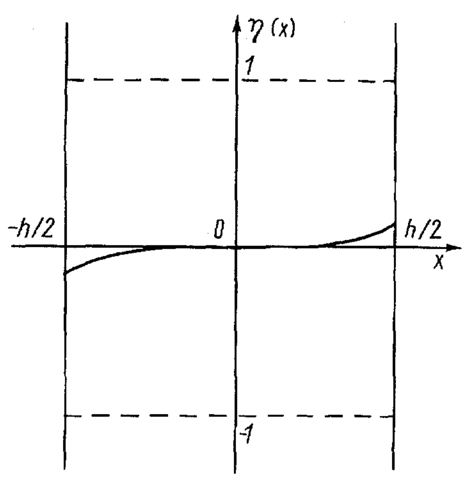
$$q(x) = eZ_+ nF_0\rho \frac{\mathrm{sn}(X;\ \lambda)\,\mathrm{dn}(X;\ \lambda)}{\mathrm{cn}^2(X;\ \lambda)}, \tag{10}$$

while the local numerical density of particles $n(x) \equiv e\sum_\alpha n_\alpha F_\alpha(x)$ is

$$n(x) = nF_0\left(2\frac{\mathrm{dn}^2(X;\ \lambda)}{\mathrm{cn}^2(X;\ \lambda)} - 1\right). \tag{11}$$

Integral equation (3) and the normalization condition

$$h^{-1}\int_{-h/2}^{h/2} n(x)\,dx = n, \tag{12}$$



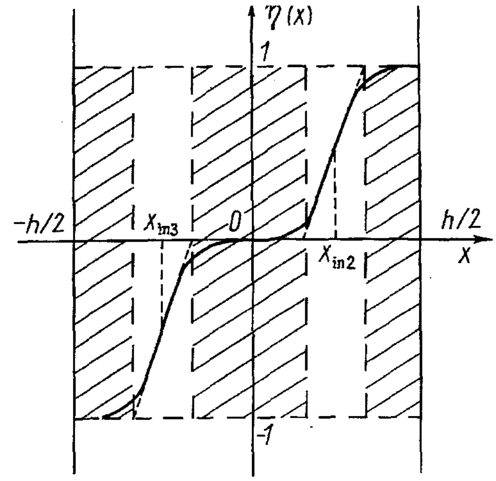


Fig. 1. Mean charge distribution of a particle in a weak external field ($eZ_+ED^*/kT < 1.22\text{th}^{-1}(h/2D)$); the plasma is almost electroneutral throughout the entire space.

Fig. 2. Mean charge distribution of a particle in a moderate external field ($1.22\text{th}^{-1}(h/2D) < eZ_+ED^*/kT < 0.71\text{ch}(h/2D)$); the plasma has a three-layer structure.

stemming from the constancy of the number of particles allow us to calculate the parameters F_0 and ρ entering (10), (11). Substitution of (10) into (3) yields [9]

$$\rho = \frac{eZ_+ED^*}{kT}\,\text{cn}\,(H;\ \lambda), \tag{13}$$

where $H \equiv h/2D^*$. The expression (11) in combination with the normalization condition (12) gives

$$F_0 = \left(1 + 2\,\frac{\dfrac{\text{sn}\,(H;\ \lambda)\,\text{dn}\,(H;\ \lambda)}{\text{cn}\,(H;\ \lambda)} - E\,(\iota;\ \lambda)}{H}\right)^{-1}, \tag{14}$$

where $E(\iota; \lambda)$ is the normal elliptic second-kind Legendre integral [10]; $\iota \equiv \arcsin\,[\text{sn}(H; \lambda)]$. Relations (9), (13), (14) completely determine the parameters F_0, ρ, λ and along with them the charge density (10).

3. We analyze the behavior of charge density (10) in some limiting cases. If the field E is small, i.e.,

$$eZ_+ED/kT \ll 1, \tag{15}$$

then from (13) and (9) we obtain $\lambda^2 \approx 1$, whence $F_0 \approx 1$ and from (10) we have

$$q\,(x) = eZ_+n\,\frac{eZ_+ED}{kT}\,\frac{\text{sh}\,\dfrac{x}{D}}{\text{ch}\,\dfrac{h}{2D}}, \tag{16}$$

which coincides with the result in [11]. The same result (16) is obtained also in the case when the plasma layer is wide, i.e.,

$$h/D \gg 1, \tag{17}$$

which is consistent with [12]. Thus, weak inhomogeneity of plasma (a linear regime) develops in a weak field and/or in its wide layer.

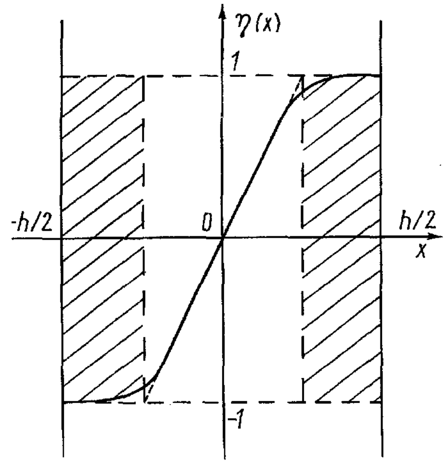


Fig. 3. Mean charge distribution of a particle in a strong external field ($eZ_+ED^*/kT > 0.71ch(h/2D)$); degeneracy of the near-electroneutral plasma layer; the two-layer plasma.

A thin layer of plasma ($\gamma < h/D < 1$) in a sufficiently strong field E ($eZ_+ED^*/kT > 2$) is highly inhomogeneous. In this regime, expressions (8), (10), (11) are not applicable because condition (7) is violated, the magnitude of elliptic integral (9) becomes an imaginary number, and the result of integration of Eq. (6), not given here for reasons of its cumbersome form, differs from (8) but coincides with it at $\lambda^2 = 0$.

4. The main specific feature of the system under consideration is violation of the quasi-neutral condition in the surface layer of plasma and structurization of the plasma layer. To describe these processes, we investigate local electrization $\eta(x)$ determined by the following expression [3]

$$\eta(x) \equiv \frac{q(x)}{eZ_+n(x)} = \frac{f^2(x) - 1}{f^2(x) + 1} \tag{18}$$

and having the meaning of the mean charge of a particle at the distance x from the plane X = 0. It is evident that in the middle plane X = 0 the local electrization is $\eta(0) = 0$. Near the plane boundary, provided the field E is sufficiently strong and $f(h/2) >> 1$, the electrization is close to unity (in absolute value), which is indicative of formation here of a practically one-species plasma layer [3]. In the intermediate region, $\eta(x)$ monotonically increases (decreases) as far as the point x approaches the boundary; however the local electrization variation has extrema within the space. The condition of inflection $\eta''(x_{in}) = 0$ with an account of (6) and the Poisson equation [13] acquires the form

$$(f^2(x_{in}) - 1)\left(f^2(x_{in}) - \frac{4}{3}[2 - \rho^2] f(x_{in}) + 1\right) = 0. \tag{19}$$

The square trinominal in (19) has real nonnegative roots if

$$\rho^2 \leqslant \frac{1}{2}. \tag{20}$$

Hence the product of these roots, as follows from (19), is equal to unity, then by virtue of (4) and (5) the points of inflection corresponding to these roots are symmetric relative to the point x = 0. Thus, if condition (20) is fulfilled, then the local electrization $\eta(x)$ has three points of inflection: $x_{in1} = 0$ and $x_{in2} = -x_{in3}$; here the minimum rate of $\eta(x)$ variation corresponds to the point x = 0, while the maximum rate is appropriate for the two other points.

In the linear regime (15), (17) $f(h/2) < f(x_{in2})$, the points x_{in2} and x_{in3} do not land in the space occupied by the plasma, and $\eta(x)$ will have, in fact, only one bending at the point x = 0. In this case, the layer of near-electroneutral plasma occupies the entire space (Fig. 1).

In the intermediate regime of moderate inhomogeneity $f(h/2) > f(x_{in2})$, and the points x_{in2} and x_{in3} are within the space occupied by plasma. This condition with an account of (8), (9), (13), (14), (19), (20) acquires the form of the following double inequality:

$$\frac{1}{3}(1+\sqrt{7})\,\mathrm{cth}\left(\frac{h}{2D}\right)\leqslant\frac{eZ_+ED^*}{kT}\leqslant\frac{1}{\sqrt{2}}\,\mathrm{ch}\left(\frac{h}{2D}\right) \tag{21}$$

which is meaningful if

$$\frac{h}{D}\geqslant 2\,\mathrm{Arsh}\left(\frac{\sqrt{2}}{3}(1+\sqrt{7})\right)\approx 2{,}62. \tag{22}$$

In this regime in the neighborhood of boundaries a layer of almost one-component plasma develops (Fig. 2), while in the central region a finite-width layer of practically electroneutral plasma is preserved. The points of intersection of the tangents to the curve $\eta(x)$ at the points of inflection x_{in2} and x_{in3} with the straight lines $\eta = 1$ and $\eta = -1$ may naturally be considered in that case as the boundaries of the corresponding surface layers of the practically one-component plasma, and the points of intersection of these tangents with the 0x axis as the boundaries of the layer of near-electroneutral plasma. Condition (21) restricts the field E both from above and from below, while inequality (22) restricts the distance between the plasma boundaries h from below. Thus, the three-layer structure of plasma exists in a not too narrow capacitor in a limited intensity range of the field E.

In the strong-inhomogeneity regime, when condition (20) is disturbed, the curve $\eta(x)$ has the single point of inflection $x = 0$ corresponding to the maximum variation of local electrization, and, consequently, the finite-width layer of near-electroneutral plasma degenerates into an infinitely thin plane, while near the boundaries the layer of practically one-component plasma is completely formed (Fig. 3).

NOTATION

T, absolute temperature; E_c, absolute value of intensity of a self-sustained electric field; γ, plasma parameter; D_l, local Debye radius; Z_α, charge multiplicity of particles of the species ; n_α, numerical density of particles of the species α; $F_\alpha(1)$, one-particle distribution function of particles of the species α in the correlation-free approximation; E, intensity of the external electrostatic field; h, thickness of a plasma layer; f(), reduced one-particle distribution function; D^*, modified Debye radius; q(x), local charge density; $\eta(x)$, local electrization.

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